Pre-Calc
Review
Section 9.4/5 – Sequences and Series

Name ________________________________

For questions 1 and 2, find the first four terms and the 100th term of the explicitly defined sequences.

1) $g_n = 2n^2 - 1$

$g_1 = 1$
$g_2 = 7$
$g_3 = 17$
$g_4 = 31$
$g_{100} = 19999$

2) $c_n = \dfrac{n^2+1}{2}$

$c_1 = 1$
$c_2 = 2.5$
$c_3 = 5$
$c_4 = 8.5$
$c_{100} = 5000.5$

For questions 3 – 6, find the first 4 terms and the 15th term of the recursively defined sequences.

3) $a_1 = 6;\ a_n = a_{n-1} - 5\ \ for\ \ n \ge 2$

$a_1 = 6$
$a_2 = 1$
$a_3 = -4$
$a_4 = -9$

$a_{15} = -64$

4) $a_1 = -3;\ a_{n+1} = a_n + 10\ \ for\ \ n \ge 1$

$a_1 = -3$
$a_2 = 7$
$a_3 = 17$
$a_4 = 27$

$a_{15} = 137$

5) $a_1 = .75;\ a_n = (-3)a_{n-1}\ \ for\ \ n \ge 2$

.75
-2.25
6.75
-20.25

3587226.75

6) $a_1 = -2;\ a_2 = -3;\ \ a_n = a_{n-2} + a_{n-1},\ \ for\ \ n \ge 3$

$a_1 = -2$
$a_2 = -3$
$a_3 = -5$
$a_4 = -8$
-13
-21
-34
-55
-89
-144
-233
-377
-610
-987
$a_{15} = -1597$

For questions 7 – 10, determine whether the sequence is arithmetic or geometric and find:

a) The common difference or ratio
b) The 10th term
c) An explicit rule for the *n*th term
d) A recursive rule for the *n*th term

7) -7, 14, 35, 56, …

a) $d = 21$
b) 182
c) $a_n = -7 + (n-1)21$
$= 21n - 28$
d) $a_n = a_{n-1} + 21$
$a_1 = -7$, $n \geq 2$

8) 3, 9, 27, 81,…

a) $r = 3$
b) 59049
c) $a_n = 3 \cdot 3^{n-1}$
d) $a_n = 3 \cdot a_{n-1}$
$a_1 = 3$, $n \geq 2$

9) -4, 4, -4, 4, …

a) $r = -1$
b) 4
c) $a_n = (-4)(-1)^{n-1}$
d) $a_1 = -4$
$a_n = (-1)a_{n-1}$, $n \geq 2$

10) 6, 10, 14, 18, …

a) $d = 4$
b) 42
c) $a_n = 6 + (n-1)(4)$
d) $a_1 = 6$
$a_n = a_{n-1} + 4$, $n \geq 2$

11) The fourth and seventh terms of an arithmetic sequence are -8 and 4, respectively. Find the first term and a recursive rule for the *n*th term.

$a_4 = -8$
$a_7 = 4$

$a_n = a_1 + (n-1)d$
$-8 = a_1 + (4-1)d$
$4 = a_1 + (7-1)d$

$a_1 + 3d = -8$
$-\ a_1 + 6d = -4$

$-3d = -12$
$d = 4$

$a_1 + 12 = -8$
$\boxed{a_1 = -20}$
$\boxed{a_n = a_{n-1} + 4, \ n \geq 2}$

12) The second and eighth terms of a geometric sequence are 3 and 192, respectively. Find the first term, common ratio, and an explicit rule for the *n*th term.

$a_2 = 3$
$a_8 = 192$

$a_n = a_1 r^{n-1}$

$192 = a_1 \cdot r^7$
$3 = a_1 r$
$64 = r^6$
$\pm 2 = r$

$\boxed{\begin{array}{l} a_1 = \frac{-3}{2} \\ r = -2 \\ a_n = \frac{-3}{2}(-2)^{n-1} \\ \hline a_1 = 3/2 \\ r = 2 \\ a_n = 3/2(2)^{n-1} \end{array}}$

For questions 13 – 16 write each sequence using summation notation, assuming the suggested pattern continues.

13) 2+5+8+11+…+29

$\boxed{\sum_{K=1}^{10} 2+(n-1)(3)}$

$a_n = a_1 + (n-1)d$
$29 = 2 + (n-1)3$
$27 = 3(n-1)$

14) -1+2+7+14+23+…+62

$\sum_{K=1}^{8} K^2 - 2$

15) $\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+...$

$\sum_{K=1}^{\infty} \left(\frac{1}{4}\right)^K$

16) -2+2-2+2-2+…

$\sum_{K=1}^{\infty} -2(-1)^{K-1}$

For questions 17 – 20, find the sums of the finite geometric or arithmetic series.

17) 117+110+103+…+33

$a_n = a_1 + (n-1)d$
$33 = 117 + (n-1)(-7)$
$n = 13$

$S_{13} = \frac{13}{2}(117+33)$
$\boxed{975}$

18) 111+108+105+…+27

$27 = 111 + (n-1)(-3)$
$n = 29$

$S_{29} = \frac{29}{2}(111+27)$
$\boxed{2001}$

19) 3+6+12+…+12,288

$\times 2$

$12288 = 3(2)^{n-1}$

$n = 13$

$S_{13} = \dfrac{3(1-2^{13})}{1-2}$

$\boxed{24573}$

20) $42 + 7 + \dfrac{7}{6} + ... + 42\left(\dfrac{1}{6}\right)^8$

9

$S_9 = \dfrac{42\left(1-\left(\frac{1}{6}\right)^9\right)}{1-\frac{1}{6}}$

$\approx \boxed{50.4}$

For questions 21 – 24, determine whether the infinite geometric series converges. If so, find its sum.

21) $4 + \dfrac{4}{3} + \dfrac{4}{9} + \dfrac{4}{27} + ...$

$r = \frac{1}{3}$ so it converges

$S_\infty = \dfrac{4}{1-\frac{1}{3}} = \boxed{6}$

22) $\displaystyle\sum_{h=1}^{\infty} 3\left(\frac{1}{4}\right)^h$

$r = \frac{1}{4}$ so it converges

$S_\infty = \dfrac{3/4}{1-\frac{1}{4}} = 1$

23) $6 + 3 + \dfrac{3}{2} + \dfrac{3}{4} + ...$

$r = \frac{1}{2}$ converges

$S_\infty = \dfrac{6}{1-\frac{1}{2}} = 12$

24) $\displaystyle\sum_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^n$

$r = \frac{2}{3}$ converges

$S_\infty = \dfrac{\frac{10}{3}}{1-\frac{2}{3}} = 10$